Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

6 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2224 – 7203

8 pages

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

The purpose of this question is to help a company decide whether or not they should use a new technique to make a component.

A factory produces components for a tractor. They have designed a new technique to produce one of their components that they hope will increase its useful lifespan.

They test 120 components made with the new technique and 240 with the technique they currently use. At the end of 250 hours of use, they check the components, and record whether they have no cracks, minor cracks or major cracks.

The data from the trial are given in the table.

	New technique	**Current technique**
No cracks	a	88
Minor cracks	54	96
Major cracks	b	56

In total 141 components had no cracks.

(a) (i) Show that the value of a is 53. [1]

(ii) Find the value of b. [1]

(This question continues on the following page)

(Question 1 continued)

One of the components from the trial is selected at random.

(b) Given that this component had minor cracks find the probability that it was produced by the new technique. [2]

(c) A χ^2 test for independence is performed at the 5% significance level to determine whether a component having no cracks, minor cracks or major cracks is independent of the production technique used.

(i) State the null and alternative hypotheses. [1]

(ii) Find the p-value. [2]

(iii) State the conclusion of the test in context, justifying your answer. [2]

(d) For the components in the trial that were made with the current technique, show that the proportion which developed cracks is $\frac{19}{30}$. [1]

As an alternative measure, the researchers decide to let p be the probability that a component, made with the new technique, develops cracks. They then test the following hypotheses

$$\mathrm{H}_0\colon\ p = \frac{19}{30},$$

$$\mathrm{H}_1\colon\ p < \frac{19}{30}.$$

In a randomly selected sample of 120 components made with the **new technique** let X be the number which developed cracks. The researchers assume that, under the null hypothesis, $X \sim \mathrm{B}\left(120, \frac{19}{30}\right)$.

(e) State one additional assumption that the researchers are making in choosing this distribution. [1]

(f) Use appropriate data from the trial to perform the test proposed by the researchers, at the 5% significance level. State the conclusion of the test, justifying your answer. [5]

(g) In comparison with the test in part (c), state one mathematical reason why

(i) the test in part (f) might be preferred. [1]

(ii) the test in part (f) might **not** be preferred. [1]

(This question continues on page 5)

Blank page

(Question 1 continued)

For these components, the researchers also consider the mean time taken until cracks develop. It is hoped that using the new technique will increase this value. A second trial is carried out and the times, in hours, taken for cracks to appear is recorded.

The mean time taken for cracks to appear ($\bar{t}$) and the value of s_{n-1} for each technique are given in the following table.

	Number of components tested	$\bar{t}$ **(hours)**	s_{n-1} **(hours)**
New technique	100	250.1	3.8
Current technique	200	249.1	3.8

(h) Perform an appropriate test at the 5% significance level to determine whether the new technique increases the mean time taken for cracks to appear. [7]

The company decides to go ahead with the new technique and publishes the following statement: “statistical tests show the new technique will significantly increase the time before components crack and need to be replaced”.

(i) Comment on this statement. [1]

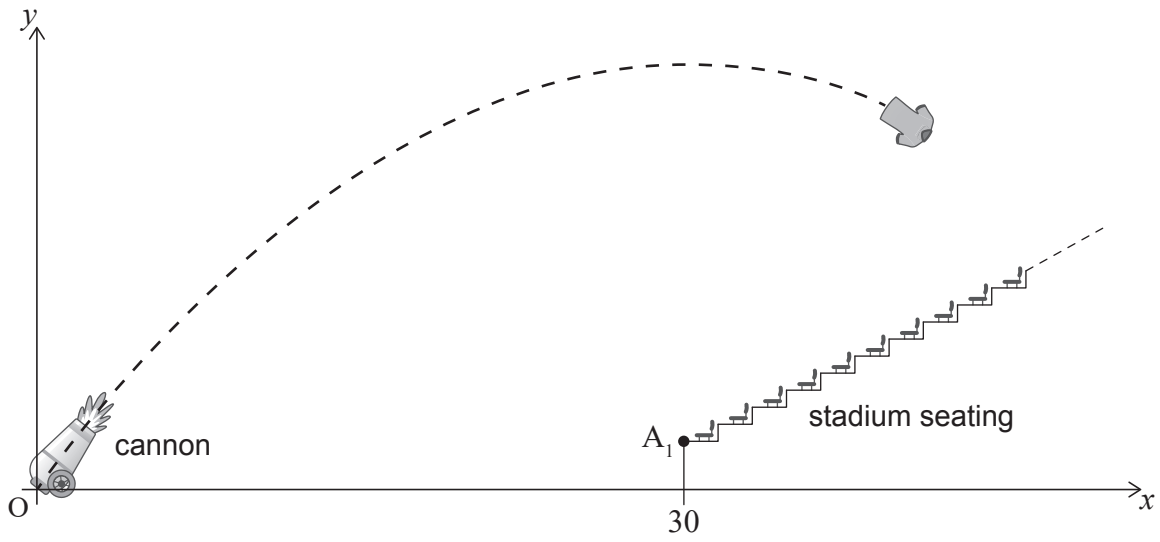
2. [Maximum mark: 29]

A sports stadium has a T-shirt cannon which is used to launch T-shirts into the crowd. The purpose of this question is to determine whether a person sitting in a particular seat will ever receive a T-shirt.

A T-shirt cannon is placed on the horizontal ground of a stadium playing area. A coordinate system is created such that the origin, O, is the point on the ground from where the T-shirts are launched. In this coordinate system, x and y represent the horizontal and vertical displacement from O, and are measured in metres.

Seat A_1 is the nearest seat to the T-shirt cannon. The coordinates of the front of the foot space for seat A_1 are $(30, 2.1)$.

diagram not to scale

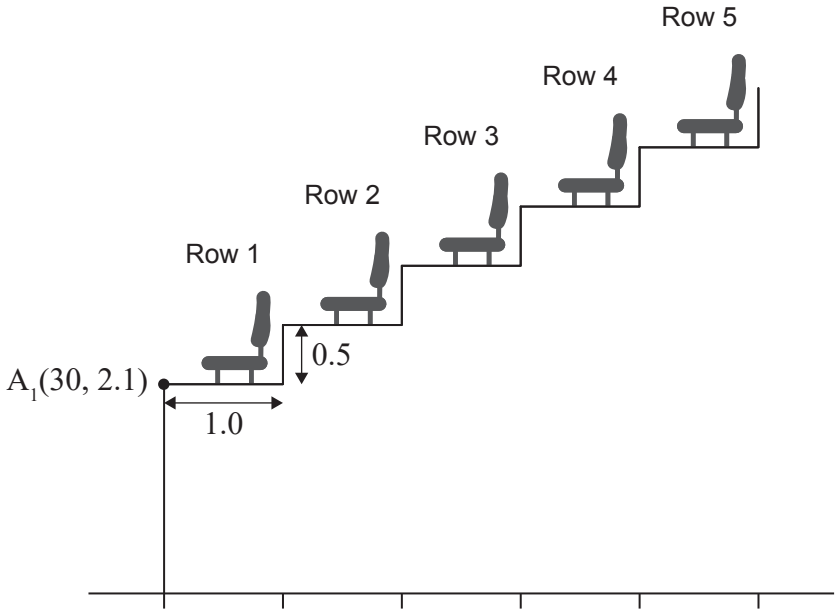


(This question continues on the following page)

(Question 2 continued)

Each seat behind seat A_1 is $1.0\,\text{m}$ further from O horizontally and $0.5\,\text{m}$ higher than the seat in the row below it, as shown on the diagram.

diagram not to scale



Seat A_1 is in row 1. Let seat A_n be the seat directly behind A_1 in row n.

(a) (i) Write down the coordinates of the front of the foot space of seat A_5. [2]

(ii) Find, in terms of n, the coordinates for the front of the foot space of seat A_n. [3]

While in motion, the T-shirt can be treated as a projectile.

Let t be the time, in seconds, after a T-shirt is launched.

At any time $t > 0$, the acceleration of the T-shirt, in m s^{-2}, is given by the vector

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}.$$

The initial velocity, in m s^{-1}, of the T-shirt is given as $\begin{pmatrix} 29.4\cos\theta \\ 29.4\sin\theta \end{pmatrix}$, where θ is the angle to the ground at which the T-shirt is launched and $0^\circ < \theta \leq 90^\circ$.

(b) (i) Find an expression for the velocity, $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$, at time t. [3]

(ii) Hence show that when the T-shirt is launched vertically, the time for it to reach its maximum height is 3 seconds. [3]

(This question continues on the following page)

(Question 2 continued)

The displacement of the T-shirt, t seconds after it is launched, is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 29.4(\cos\theta)t \\ 29.4(\sin\theta)t - 4.9t^2 \end{pmatrix}.$$

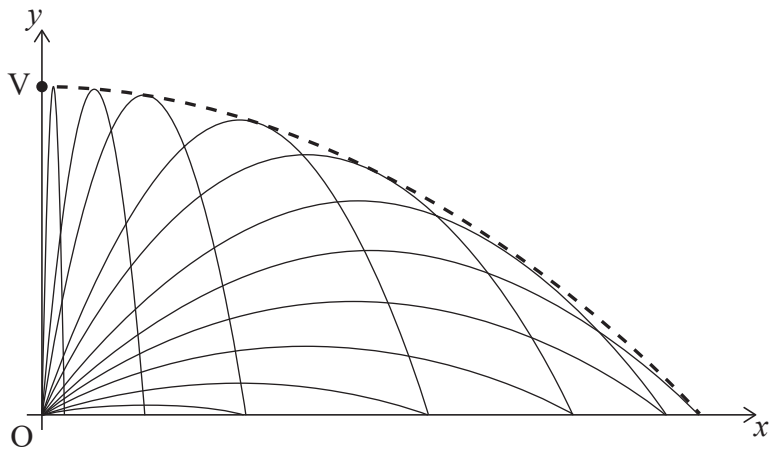
(c) Using the given answer to part (b)(ii) or otherwise, find the maximum height reached by a T-shirt when it is launched vertically. [2]

(d) (i) If there was no seating, and the T-shirt was launched at an angle θ, show that the value of x when it would hit the ground is given by the expression

$$x = 176.4\sin\theta\cos\theta.$$ [3]

(ii) Hence find the maximum possible value for x if there was no seating to block the path of the T-shirt. [2]

In order to calculate the seats in the stadium which can be reached by a T-shirt it is required to find the equation of the curve that forms the boundary of all the points that can be reached. This boundary is represented by the dashed curve in the following diagram, while the solid curves represent some of the possible trajectories for the T-shirts.



It is given that the boundary curve is the parabola $y = ax^2 + bx + c$, with its vertex V on the y-axis.

(e) Using your answers to parts (c) and (d)(ii), or otherwise, find

(i) the value of c. [1]

(ii) the value of b. [2]

(iii) the value of a. [3]

A spectator is sitting in seat A_{40}.

(f) Show that it is not possible for the spectator to ever get a T-shirt. [5]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References: