Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

25 October 2024

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

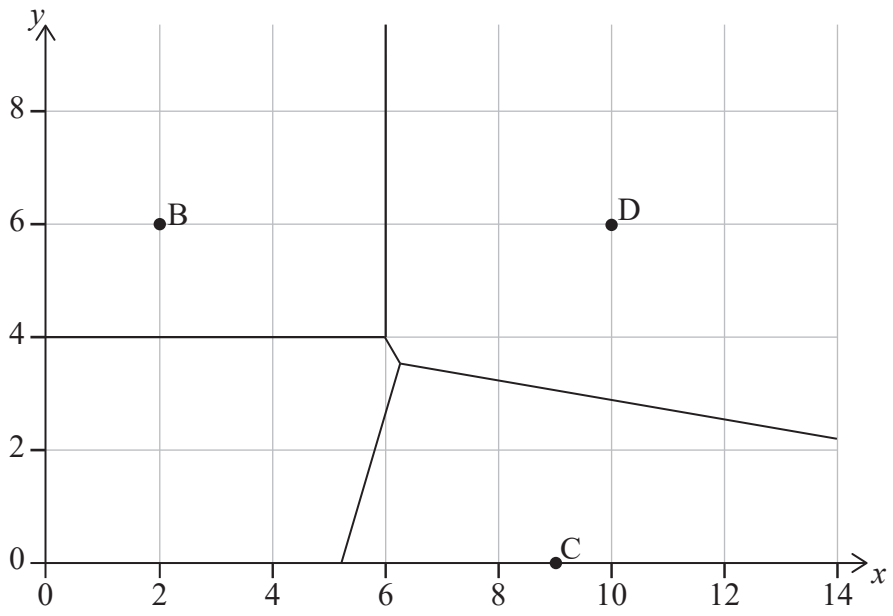
8824 – 7210

11 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points A, B, C and D. The Voronoi diagram for these four points is shown. Point A is not shown.



One unit represents 1 metre.

Point B is located at $(2, 6)$.

The equation of the perpendicular bisector of $[\mathrm{AB}]$ is $y = 4$.

(a) Write down the coordinates of A. [1]

Points C and D are located at $(9, 0)$, and $(10, 6)$, respectively.

(b) Find

(i) the coordinates of the midpoint of $[\mathrm{CD}]$

(ii) the equation of the perpendicular bisector of $[\mathrm{CD}]$. Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	A	B	C	D
Average waiting time (minutes)	9	5	10	7

(c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point $(4, 6)$. [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t-test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

(d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

(e) (i) State which one of the following methods best describes the owner's planned sampling:

convenience, systematic, or stratified.

(ii) State one disadvantage of this sampling technique. [2]

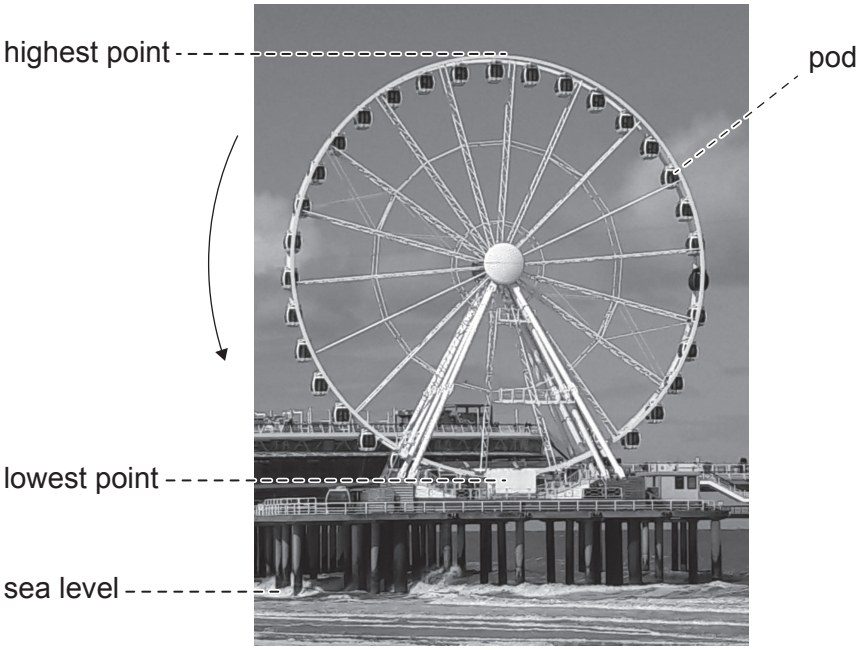
The restaurant owner instead collects the data using simple random sampling.

The p-value is 0.0897 for the sampled data.

(f) State the conclusion for the test. Justify your answer. [2]

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is $8\,\text{m}$ above sea level, and its highest point is $45\,\text{m}$ above sea level.



(a) (i) Show that the radius of the Ferris wheel is $18.5\,\text{m}$.

(ii) Calculate the circumference of the Ferris wheel. [2]

There are pods, equally spaced around the wheel, that carry passengers.

(b) When the wheel rotates 10°, find the distance that a pod travels along the circumference. [3]

(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a\sin(bt) + d, \text{ for } a, b > 0,$$

where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

(c) (i) Find the value of b.

(ii) Find the value of d.

(iii) Hence, write down the equation of the sinusoidal model. [5]

(d) Use the model to find the values of t when the height of this pod is $33\,\text{m}$ above sea level for $0 \le t \le 15$. [3]

Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

(e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer. [3]

3. [Maximum mark: 14]

A survey was answered by $20\,000$ expatriates (people living in a country that is not their own). The data ranked countries in order of the country they felt was best for expatriates. The highest-ranked country was Switzerland.

These results were compared to happiness scores taken from *The World Happiness Report 2022*. The following table shows this data for the top 10 expatriate countries.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Expatriate country rank	1	2	3	4	5	6	7	8	9	10
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4

(a) For the **happiness score**, find

(i) the upper quartile

(ii) the interquartile range. [4]

(b) Show that Switzerland's happiness score is not an outlier for this data. [3]

(This question continues on the following page)

(Question 3 continued)

The happiness scores were ranked to calculate Spearman's rank correlation coefficient, r_s. These ranks are shown in the following table.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4
Happiness rank	1	a	b	c	9	10	5	7	6	2

(c) Write down the value of

(i) a

(ii) b

(iii) c. [3]

(d) (i) Find r_s.

(ii) If France's happiness score is upgraded to 6.9, explain why the value of r_s does not change. [3]

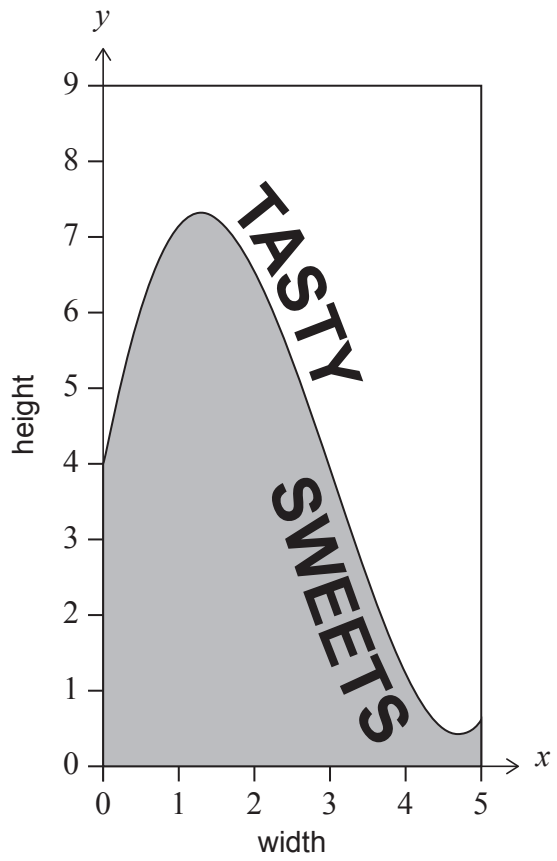
Jose concludes from this data that countries with high happiness scores are likely to be favourite expatriate countries.

(e) State, with a reason, whether Jose's conclusion is appropriate. [1]

4. [Maximum mark: 20]

Sweets are sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is $5\,\text{cm}$ wide and $9\,\text{cm}$ high.

The design on the label is a curve, as shown on the following axes, where one unit represents $1\,\text{cm}$ for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	4	7.3	6.7	4.0	1.3	0.7

(a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve. [3]

The curve used in the label design can be modelled by:

$$y = \frac{x^3}{3} - 3x^2 + 6x + 4 \text{, for } 0 \le x \le 5 .$$

(b) (i) Write down the integral that represents the area of the shaded region.

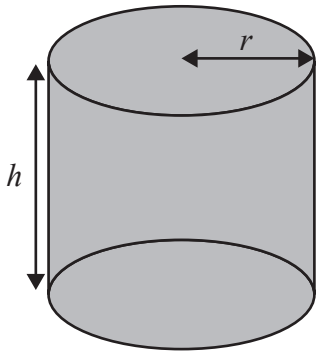
(ii) Hence, find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

The sweets are sold in closed cylindrical containers, with radius r and height h.

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of $600\,\text{cm}^3$.

(c) Write down an equation, in terms of r and h, that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A, of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

(d) Find the value of k. [4]

(e) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}r}$.

(ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

The containers are made so that the surface area is minimized. The $5\,\text{cm}$ by $9\,\text{cm}$ rectangular label is to be glued to the curved surface of the container.

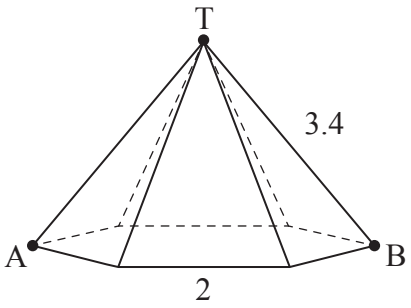
(f) Show that the label will fit on the container. [3]

5. [Maximum mark: 17]

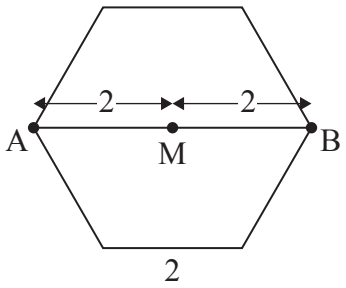
Gaurika is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre M. The length of each side of the base is $2\,\mathrm{m}$, the length of each sloping edge is $3.4\,\mathrm{m}$, and the distance between each vertex on the base and M is $2\,\mathrm{m}$, as shown in the diagrams.

diagrams not to scale

3D view of tent



2D view of base

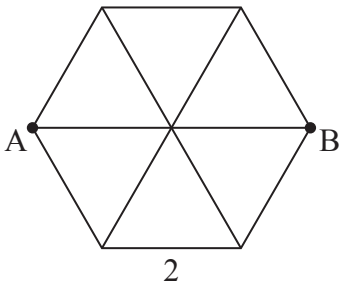


The top of the tent, T, will be supported by a vertical pole from M.

(a) Find the length of the pole, MT. [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



(b) Find

(i) the area of the base

(ii) the volume of the tent. [5]

(c) Find the value of $\mathrm{M\hat{A}T}$. [2]

(This question continues on the following page)

(Question 5 continued)

For extra support, Gaurika decides to attach a rope, with length $2.6\,\text{m}$, to the tent at a point, X, on the edge AT.

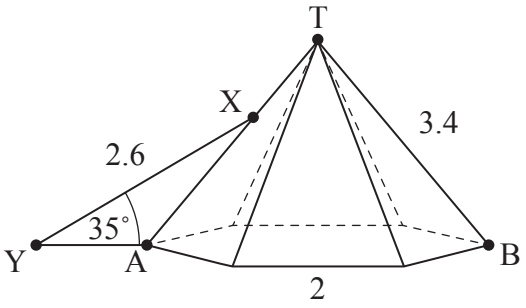
The rope will be fixed to the ground at point Y, such that:

- the rope, $[\text{XY}]$, is straight
- points Y, A and B lie on a straight line
- $\text{A}\hat{\text{Y}}\text{X} = 35^\circ$.

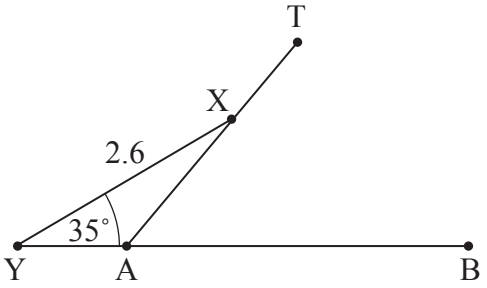
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle AXY



(d) Find AY. [4]

For decoration at night, Gaurika wants to fix a strip of lights from point A to a point, Z, along the rope $[\text{XY}]$.

The strip of lights, $[\text{AZ}]$, is straight and has length $0.9\,\text{m}$.

(e) Find the two possible values of YZ. [4]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

3. InterNations. 2022. *Expat Insider 2022*. [online] Available at: https://www.internations.org/expat-insider/2022/. Source adapted.

Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for "Happiness, benevolence, and trust during COVID-19 and beyond," Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/ [Accessed 13 November 2023]. Source adapted.