

Let $z = a + bi$

Find a and b if $z^2 = |z|^2 - 4$

$$z^2 = (a + bi)(a + bi)$$

$$z^2 = a^2 + abi + abi + b^2i^2$$

$$z^2 = a^2 - b^2 + 2abi$$

Use Pythagoras' Theorem to find $|z|$

$$|z|^2 = a^2 + b^2$$

$$z^2 = |z|^2 - 4$$

$$a^2 - b^2 + 2abi = a^2 + b^2 - 4$$

We can

- equate the real parts
- equate the imaginary parts

$$a^2 - b^2 = a^2 + b^2 - 4$$

$$-b^2 = b^2 - 4$$

$$2b^2 = 4$$

$$b = \pm\sqrt{2}$$

$$a = 0$$

Since $2ab = 0$