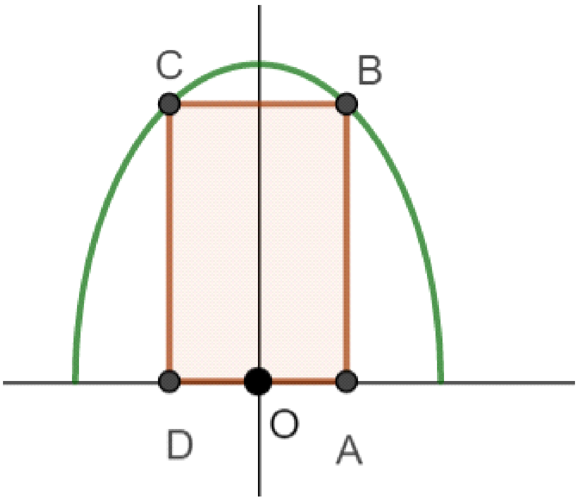
A rectangle ABCD is drawn so that its lower vertices are on the x-axis and its upper vertices are on the curve

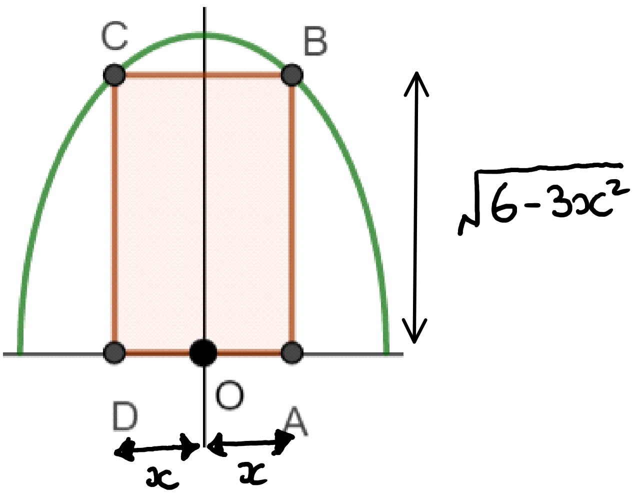
$f(x) = \sqrt{6 - 3x^2}$ as shown in the following diagram



Let $OA = x$
The area of this rectangle is denoted by A.

(a) Write down an expression for A in terms of x.

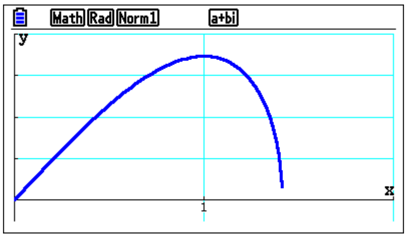
(b) Find the maximum value of A.



Width $= 2x$
Height $= \sqrt{6 - 3x^2}$

$$A = 2x\sqrt{6 - 3x^2}$$

Minimum area occurs when $\frac{dA}{dx} = 0$



$$A = 2x\sqrt{6 - 3x^2}$$

This is a product so we need to use the Product Rule

$A = uv$

$\frac{dA}{dx} = u\frac{dv}{dx} + \frac{du}{dx}v$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$v = (6 - 3x^2)^{\frac{1}{2}}$$

$$\frac{dv}{dx} = \frac{1}{2}(-6x)(6 - 3x^2)^{-\frac{1}{2}}$$

$$\frac{dv}{dx} = \frac{-3x}{\sqrt{6 - 3x^2}}$$

$$\frac{dA}{dx} = 2x\frac{-3x}{\sqrt{6 - 3x^2}} + \sqrt{6 - 3x^2} \cdot 2$$

Solve $\frac{dA}{dx} = 0$

$$\frac{-6x^2}{\sqrt{6-3x^2}} + 2\sqrt{6 - 3x^2}=0$$

$$2\sqrt{6 - 3x^2}=\frac{6x^2}{\sqrt{6-3x^2}}$$

$$6 - 3x^2 = 3x^2$$

$$6 = 6x^2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$A = 2x\sqrt{6 - 3x^2}$$

$$A = 2\sqrt{6 - 3}$$

$$A = 2\sqrt{3}$$