Let the function f be defined by

$$f(x) = \frac{x-k}{2-x}\ , x \neq 2$$

a) Find $f^{-1}(x)$ in terms of ***k***

b) Find ***k,*** if $y = f(x)$ and $y = f^{-1}(x)$ intersect at the points (-2,-2) and (3,3),

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a) $$f(x) = \frac{x-k}{2-x}\ , x \neq 2$$

Since there is a horizontal asymptote at y = -1

Range of ***f*** is $f(x) \in \mathbb{R}, f(x) \neq -1$

$$y = \frac{x-k}{2-x}$$

Interchange x and y

$$x = \frac{y-k}{2-y}$$

Make y the subject

$$\mathrm{x(2-y) = y - k}$$

$$2x - xy = y - k$$

$$2x + k = y + xy$$

$$2x + k = y(1+x)$$

$$\frac{2x+k}{1+x} = y$$

$$f^{-1}(x) = \frac{2x+k}{1+x}\ , \qquad x \neq -1$$

b) $y = f(x)$ and $y = f^{-1}(x)$ intersect when

$$\frac{x-k}{2-x} = \frac{2x+k}{1+x}$$

Solve for ***x***

$$(x-k)(1+x) = (2x+k)(2-x)$$

$$x + x^2 - k - kx = 4x - 2x^2 + 2k - kx$$

INTHINKING

$3x^2 - 3x - 3k = 0$

$x^2 - x - k = 0$

If $y = f(x)$ and $y = f^{-1}(x)$ intersect at the points (-2, -2) and (3,3), then

$x = 2$ and $x = -3$ are roots of the quadratic equation

Hence

$a(x-2)(x+3) = 0$ and $a = 1$

$(x-2)(x+3) = 0$

$x^2 + x - 6 = 0$

Hence $k = 6$