

Find the particular solution to the differential equation $\frac{dy}{dx} = \frac{xe^x}{\cos y}$ if $y(0) = \frac{\pi}{2}$

$$\frac{dy}{dx} = \frac{xe^x}{\cos y}$$

$$\int \cos y \, dy = \int xe^x \, dx$$

Integration by Parts $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

$$\sin y = xe^x - \int e^x \, dx$$
$$u = x \quad \frac{du}{dx} = e^x$$
$$\sin y = xe^x - e^x + C$$
$$\frac{du}{dx} = 1 \quad v = e^x$$

when $x=0, y=\frac{\pi}{2}$

$$\begin{aligned}\sin \frac{\pi}{2} &= 0 \cdot e^0 - e^0 + C \\ 1 &= 0 - 1 + C \\ C &= 2\end{aligned}$$

$$\sin y = xe^x - e^x + 2$$

$$\sin y = e^x(x-1) + 2$$

Give your answer in the form $y = f(x)$

$$y = \arcsin(e^x(x-1)+2)$$