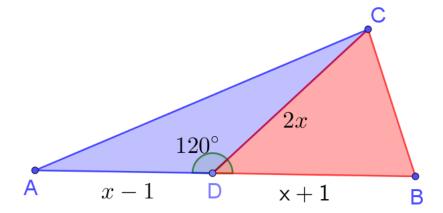
The following diagram shows a quadrilateral ABCD.



AD = x
$$-$$
 1 , BD = x $+$ 1 , DC = 2x and $\angle ADC = 120^{\circ}$

The sum of the area of triangle ADC and triangle BDC is $4\sqrt{3}$

Find x

Area of triangle ADC
$$= \frac{1}{2} \times 2x(x-1) \times sin120^{\circ}$$

$$= x(x-1) \times \frac{\sqrt{3}}{2}$$
 Area of triangle BDC
$$= \frac{1}{2} \times 2x(x+1) \times sin60^{\circ}$$

$$= x(x+1) \times \frac{\sqrt{3}}{2}$$

Sum of areas = $4\sqrt{3}$

Factorise
$$\frac{\sqrt{3}}{2}x(x-1) + \frac{\sqrt{3}}{2}x(x+1) = 4\sqrt{3}$$

$$\frac{\sqrt{3}}{2}x\{(x-1) + (x+1)\} = 4\sqrt{3}$$

$$\frac{\sqrt{3}}{2}x(2x) = 4\sqrt{3}$$

$$\sqrt{3}x^2 = 4\sqrt{3}$$

$$x^2 = 4$$

$$x = 2$$