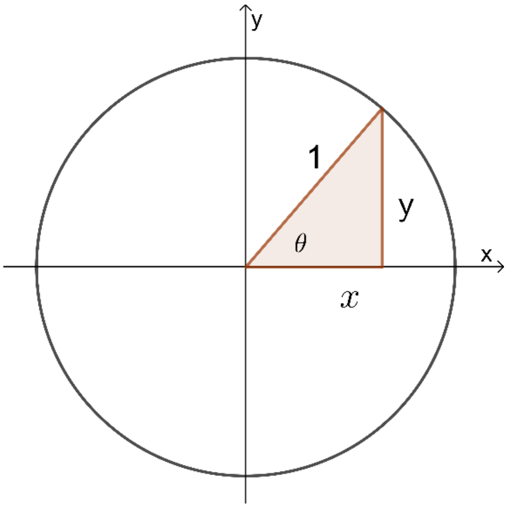
Pythagorean Trig Identities



From the unit circle, we know that

$$\cos\theta = \frac{x}{1}$$

$$\sin\theta = \frac{y}{1}$$

As this is a right-angled triangle, we can use Pythagoras' Theorem

$x^2 + y^2 = 1$

$$\cos^2\theta + \sin^2\theta = 1$$

As this is true for **ALL** angles, it is an identity and we should use the 'identically equal to' symbol rather than 'equal to'

We should write

$$\boldsymbol{\cos^2\theta + \sin^2\theta \equiv 1}$$

We often rearrange this when we use it

$$\boldsymbol{\cos^2\theta \equiv 1 - \sin^2\theta}$$

$$\boldsymbol{\sin^2\theta \equiv 1 - \cos^2\theta}$$

We can derive two more identities that use $\boldsymbol{\tan\theta}$, $\boldsymbol{cosec\theta}$, $\boldsymbol{\sec\theta}$ and $\boldsymbol{\cot\theta}$

$$\boldsymbol{\cos^2\theta + \sin^2\theta \equiv 1}$$

Divide both sides by $\cos^2\theta$

$$\boldsymbol{\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}}$$

$$\boldsymbol{1 + \tan^2\theta \equiv \sec^2\theta}$$

$$\boldsymbol{\cos^2\theta + \sin^2\theta \equiv 1}$$

Divide both sides by $\sin^2\theta$

$$\boldsymbol{\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}}$$

$$\boldsymbol{\cot^2\theta + 1 \equiv cosec^2\theta}$$

Hence, we have 3 identities

$$\boldsymbol{\cos^2\theta + \sin^2\theta \equiv 1}$$

$$\boldsymbol{1 + \tan^2\theta \equiv \sec^2\theta}$$

$$\boldsymbol{1 + \cot^2\theta \equiv cosec^2\theta}$$