HL

A plane Π has vector equation $r = \begin{pmatrix}1\\2\\0\end{pmatrix} + \mu\begin{pmatrix}-2\\0\\5\end{pmatrix} + \lambda\begin{pmatrix}0\\-4\\5\end{pmatrix}$

a) Find the Cartesian equation of the plane Π

b) The plane Π meets the x, y and z axes at A, B and C respectively. OABC forms a pyramid. Find the volume of the pyramid.

a) Normal to plane is perpendicular to $\begin{pmatrix}-2\\0\\5\end{pmatrix}$ and $\begin{pmatrix}0\\-4\\5\end{pmatrix}$

$$\begin{pmatrix}-2\\0\\5\end{pmatrix} \times \begin{pmatrix}0\\-4\\5\end{pmatrix} = \begin{pmatrix}0\cdot5 - 5\cdot(-4)\\ -((-2)\cdot5 - 5\cdot0)\\ (-2)\cdot(-4) - 0\cdot0\end{pmatrix}$$

$$= \begin{pmatrix}20\\10\\8\end{pmatrix} = 2\begin{pmatrix}10\\5\\4\end{pmatrix}$$

Check if perpendicular

$$\begin{pmatrix}10\\5\\4\end{pmatrix}\cdot\begin{pmatrix}-2\\0\\5\end{pmatrix} = 0 \qquad \begin{pmatrix}10\\5\\4\end{pmatrix}\cdot\begin{pmatrix}0\\-4\\5\end{pmatrix} = 0$$

scalar products $= 0 \therefore \perp$

$(1,2,0)$ is a point in the plane

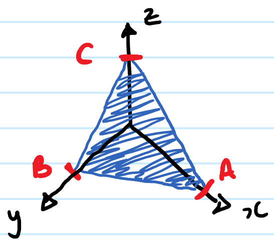
$$r \cdot n = a \cdot n$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}10\\5\\4\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}10\\5\\4\end{pmatrix}$$

$$10x + 5y + 4z = 1\cdot10 + 2\cdot5 + 0\cdot4$$

$$10x + 5y + 4z = 20$$

b) It is useful to draw a sketch



Crosses x axis when $y=0, z=0$

$10x + 0 + 0 = 20$

$x = 2$

$A(2,0,0)$

Crosses y axis when $x=z=0$

$0 + 5y + 0 = 20$

$y = 4$

$B(0,4,0)$

Crosses z axis when $x=y=0$

$0 + 0 + 4z = 20$

$z = 5$

$C(0,0,5)$

Volume of Pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times \frac{2\times4}{2} \times 5$

$= 5$ units^3