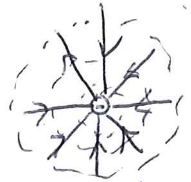
Electric fields

Fields

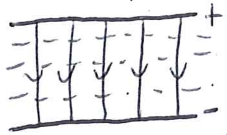
Radial

repulsive



attractive

uniform



$E = \frac{V}{d}$ (Vm^{-1})

NB: vector: $E = \frac{F_E}{q} = \frac{kQ}{r^2}$ (NC^{-1})

q → small positive test charge

Coulomb's Law → $F_E = \frac{kQq}{r^2}$

$k = \frac{1}{4\pi\varepsilon_0}$

On a surface: $\frac{Q}{A} = \varepsilon_0 E$

permittivity of free space

potential difference

$\Delta V = \frac{\Delta W}{Q}$

ΔW → work done

Q → charge

Potential

$\Delta W = Q(V_2 - V_1)$

$\Delta V = \frac{F \Delta r}{Q}$

F ← force

Δr ← distance

$V = \frac{kQ}{r}$

$\frac{\text{potential energy}}{\text{charge}}$

NB: all scalar! (add everything!)

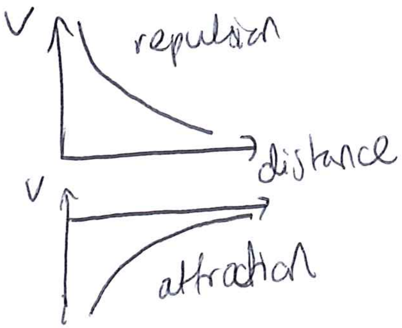
Energy → $E_p = QV$

Potential gradient

$\frac{\Delta V}{\Delta r} = -E$

Uniform

Radial



Gravitational Force

N's Law: $F_G = \frac{\overbrace{GMm}^{\text{product of bodies' masses}}}{r^2}$ ← separation distance

universal gravitational constant $= 6.667\times10^{-11}\,Nkg^{-2}m^2$

- F_G attractive (always!)
- F_G is in line with the line joining bodies
- F_G is vector

Links to Kepler's Third Law:

$$F_G = \underbrace{\frac{GMm}{r^2}}_{\text{gravitation}} = \underbrace{\frac{mv^2}{r}}_{\text{centripetal}}$$

$\Rightarrow v^2 = \frac{GM}{r}$ BUT $v = \frac{2\pi r}{T}$

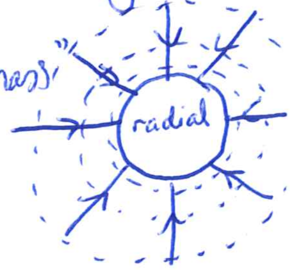
$\therefore \frac{r^3}{T^2} = \underbrace{\frac{GM}{4\pi^2}}_{\text{constant}}$

★ Can calculate the orbital radius for a geostationary satellite e.g. SATNAV

Gravitational Field Strength

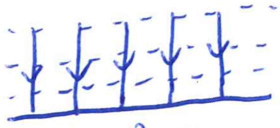
$g = \frac{F}{m}$ ← "test mass"

On Earth $\approx 9.81\,Nkg^{-1}$



Since $F = \frac{GMm}{r^2}$

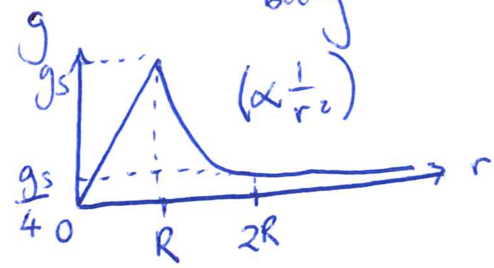
$\Rightarrow g = \frac{GM}{r^2}$ ← sphere (provided outside!) point



constant g in all locations

At the surface of a large mass, g is maximum

$g_s = \frac{GM}{R^2}$ ← radius of body



Gravitational Potential

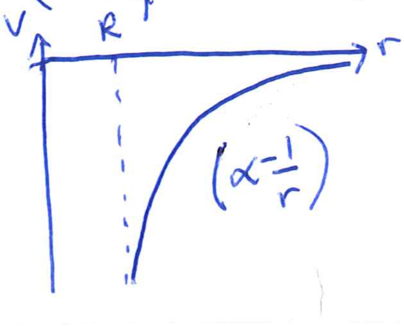
Work done per unit mass to move a small object at a slow constant speed from infinity to that point.

$\Delta W = \Delta E_p$ ← thermodynamics ☺

$E_p = -\frac{GMm}{r}$ NB: At infinity, $E_p = 0$

$(\Delta)V_g = \frac{(\Delta)E_p}{m} = \frac{(\Delta)W}{m} = \left(-\frac{GM}{r_f}\right) - \left(-\frac{GM}{r_i}\right)$

↑ scalar quantity!



Potential Gradient

A measure of how quickly gravitational potential changes with distance $= \frac{\Delta V}{\Delta r}$ NB: $\Delta W = F\Delta r$

$\Rightarrow \Delta V = \frac{F\Delta r}{m}$, $F_G = -\frac{m\Delta V}{\Delta r}$

NB: $g = \frac{F_G}{m} = -\frac{\Delta V}{\Delta r} = -$ potential gradient

For uniform fields, $E_p = mgh$

so $V = gh$

so $\frac{\Delta V}{\Delta h} = g$

Objects in Orbit

$$E_T = \underbrace{\frac{1}{2}mv^2}_{\text{kinetic energy}} - \underbrace{\frac{GMm}{r}}_{\text{gravitational potential energy}}$$

E_T ↑ total energy

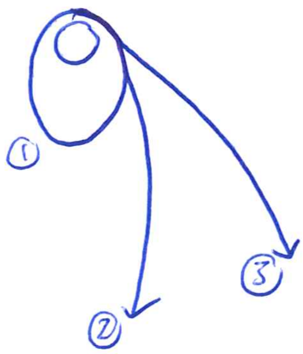
Since $F = \frac{mv^2}{r} = \frac{GMm}{r^2}$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

NB: $v = r\omega$

So $E_T = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$

if $E_K + E_P < 0$ (i.e. $E_T < 0$), circle/ellipse ①
if $E_T = 0$, parabola ②
if $E_T > 0$, hyperbola ③



Escape Velocity

Launch speed v_{esc} when body reaches infinity (i.e. $E_T = 0$)

Since $\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

since $g = \frac{GM}{R^2}$, $v_{esc} = \sqrt{2gR}$

However, other forces:
- friction with atmosphere
- gravitational pull of other massive objects

NB: Ballistic motion only (no rocket fuel!)